$$f(\tfrac{1}{2}\pi)-f(0)=\int_0^{\frac{1}{2}\pi}\frac{xdx}{\sin x(1+.16\cos^2 x)^{\frac{3}{2}}}=\frac{1728M_3-23328M_2+233280M_1}{211,680}$$
$$=1.657,636,33...(8).$$

We may check the accuracy of the work of computation and determine the degree of approximation attained in (8) by finding the values of $f'(\frac{\pi}{8})$ and $f'(\frac{3\pi}{8})$. This done we may take $m_1=6$, $m_3=4$, $m_2=3$. Substituting these values of m_1, m_2, m_3, in (7) we get an expression similar to (8) which gives a value of the definite integral coinciding with the value found in (8) for six decimal places, thus proving the accuracy of the work and showing the number of decimal places to which result found by (8) is correct.

This result is not as accurate as that obtained in the April MONTHLY for the reason that the *smallest* value of m here used is 2. This is therefore a more accurate result than would have been obtained by taking $m=2$ and developing by (1) so far as to include the term involving B_4, but a less accurate result than was obtained in the MONTHLY by taking $m=6$ and developing by (1) far enough to include the term involving B_3. The above result is, however, as accurate as Dr. Hill's and was obtained with much less labor. To obtain a more accurate result than that given in the MONTHLY without finding any higher derivatives than are there given, take $i=4$, $r=1$, $m_1=6$, $m_2=3$, and substitute in (6). By using nine decimals throughout the result is found to be 1.657,636,259. By using ten decimals throughout a result correct to nine or ten decimal places would have been obtained.

APPROXIMATION OF THE GREATEST ROOT OF A CUBIC EQUATION WITH THREE REAL ROOTS.

By CHARLES GILPIN, JR., Philadelphia, Pa.

We are concerned with the "irreducible case," in which Cardan's formula is of no value for computation. By replacing x by $-x$ if necessary, we need consider only the form

$$x^3-ax-b=0 \quad (a \text{ and } b \text{ positive}) \quad ...(1),$$

the two lesser roots of which are negative and the greatest root, which equals the sum of the lesser ones, is positive. It can be shown* that the greatest root g lies between the limits $\sqrt{a}$ and $\sqrt{(4a/3)}$. From equation (1) we obtain

*$g>\sqrt{a}$ by (2). Since $b^2/4-a^3/27<0$ in the irreducible case, x^3-ax-b is positive for $x=\sqrt{(4a/3)}$, negative for $x=\sqrt{a}$. EDITOR.

$$x = \sqrt{a + \frac{b}{x}} \ldots (2).$$

It is evident that if we assume for g an approximate value less than the true one, and substitute it in the right hand member of equation (2), the resulting value will be greater than the true value; if we assume for g a value greater than the true one, the resulting value will be less than the true value.

For a first approximation to the value of g, assume any convenient positive value between the limits $\sqrt{a}$ and $\sqrt{(4a/3)}$. For a second approximation, substitute the first approximation in the right hand member of equation (2) and compute the resulting value; etc.

This will give a series of approximations, alternately less and greater than the true value, towards which they converge as a limit.

Example. $x^3 - 6x - 2 = 0$.

$$\sqrt{6} = 2.449,$$

$$\sqrt{6 + \frac{2}{2.449}} = 2.610,$$

$$\sqrt{6 + \frac{2}{2.610}} = 2.601,$$

$$\sqrt{6 + \frac{2}{2.601}} = 2.6019,$$

$$\sqrt{6 + \frac{2}{2.6017}} = 2.601677,$$

the last being correct to five decimals.

ON THE FORMULA FOR THE AREA OF A CURVE IN POLAR CO-ORDINATES.

By JACOB WESTLUND, Purdue University.

In deriving the formula $A = \frac{1}{2}\int_{\theta_1}^{\theta_2} \rho^2 d\theta$ for the area between a curve and two radii vectores it is customary to consider the area as the limit of the sum of infinitesimal circular sectors. This formula* may, however, be derived directly from the formula $A = \int_{x_1}^{x_2} y dx$ for the area between a curve, the axis of x, and two

*It is very probable that this desirable method of proof occurs in the literature; it has been in use in Professor E. H. Moore's course in Calculus. Ed. D.